Lec 7: 09/18/2013

Dynamical Solution to the Flatness and Isotropy Problems:

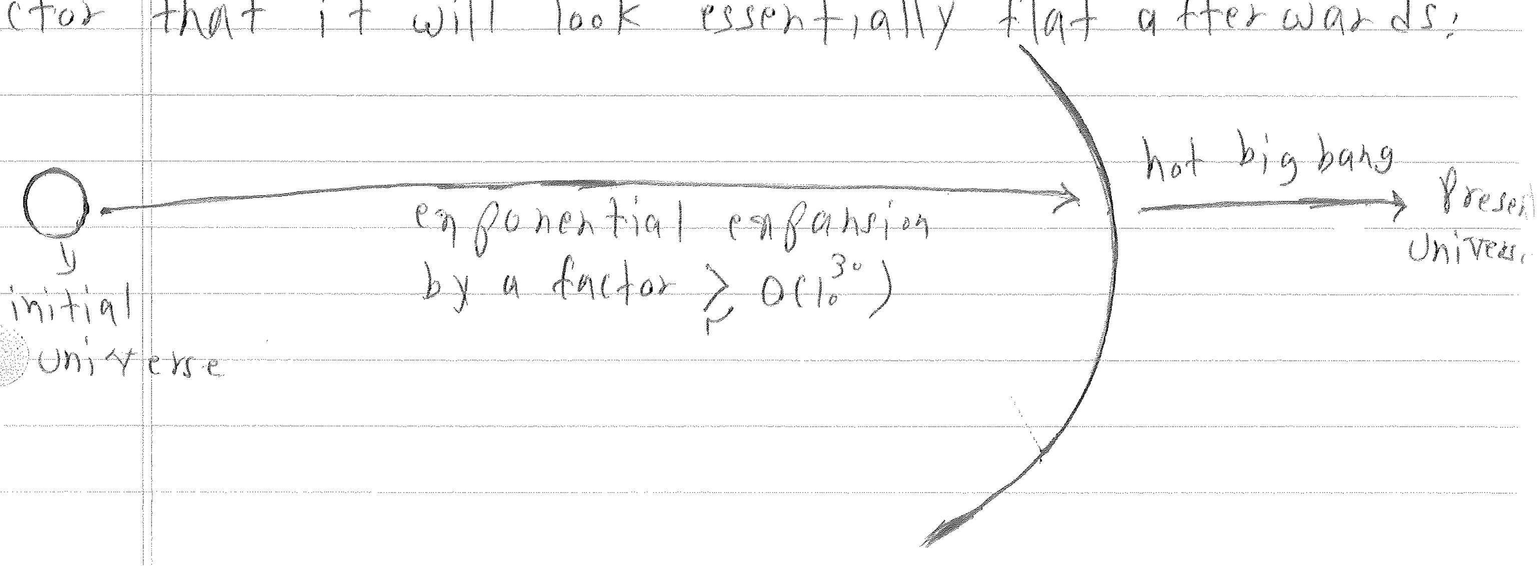
As we discussed in the last lecture, the hot big-bang model suffers from some aesthetic problems. Two notable such problems are the "flatness" and "isotropy" problems. In order for the present universe look almost flat, it must have been initially large. The amount that curvature contributed to the expansion rate right after the big-bang must have been $< O(10^{-60})$. Moreover, all different regions in such a big universe must have had the same temperature (at the level of 1 in 10^5) in order to explain the isotropy in the CMB temperature.

Both of these requirements can be met by proper initial conditions. But this requires severe fine tuning, which presents us with an unattractive feature. The question is whether both problems can be solved in a dynamical manner. Such a solution would require a modification of the big bang model in which initial deviations from the flatness and isotropy were not extraordinarily small.

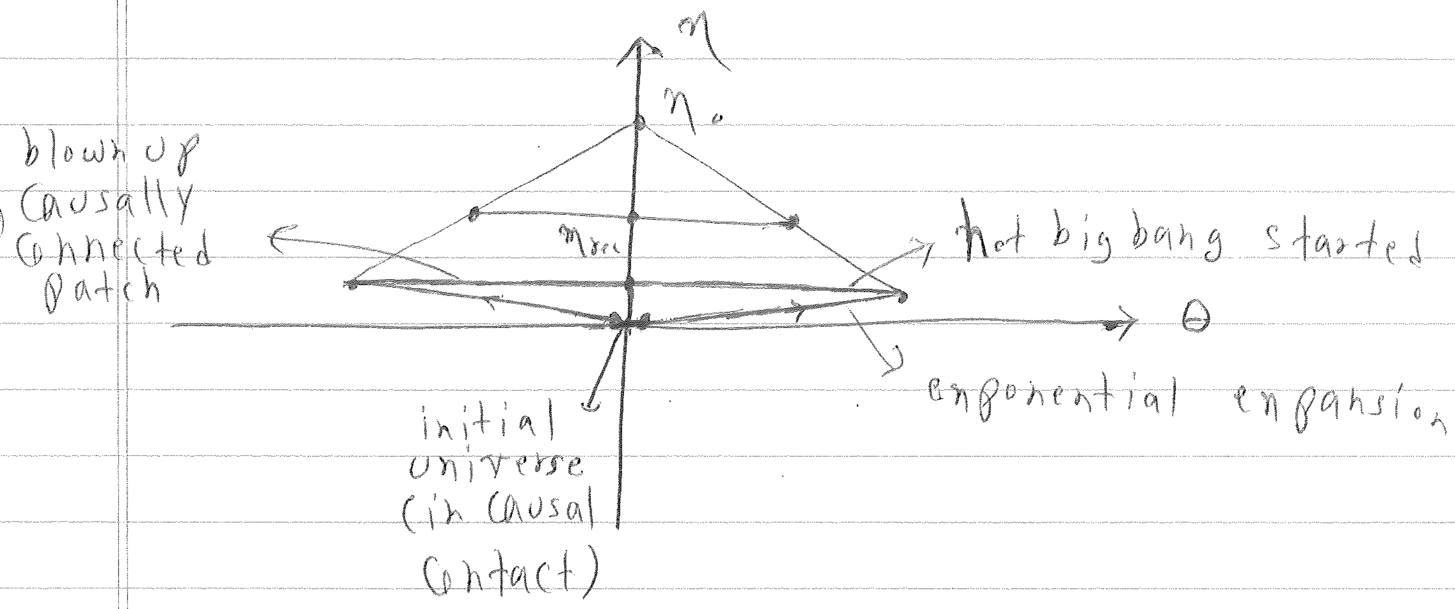
Both of these problems originate from the fact that Ha is a decreasing function of time in a universe with decelerating expansion (whis is the case in the hot big bang model). This can be easily seen in a radiation dominated universe ($H \propto t^{-1}$, $a_{(t)} \propto t^{\frac{1}{2}}$) and in a matter dominated universe ($H \propto t^{-1}$, $a_{(t)} \propto t^{\frac{2}{3}}$). Regarding the flatness problem ($\Omega = 1$) deviation from a geometrically flat universe is given by $\frac{1}{H^2 a^2}$, which necessitates an extremely large value of Ha right after the big bang. Regarding the isotropy problem, the size of a causally connected region relative to the size of the universe is $\frac{1}{Ha}$, which implies the universe had consisted of a large number of causally disconnected regions right after the big bang.

An early phase of accelerated expansion can solve both of these problems. Since $H^2a^2 = \dot{a}^2$, during an accelerated expansion Ha increases in time. As a result, $\frac{1}{H^2a^2}$ can substantially decrease when expansion is accelerating. One may then consider a situation where $Ha \sim O(1)$ initially. A brief period of accelerated expansion then decreases Ha by a factor $\gtrsim O(10^{30})$. This phase ends and the universe enters a standard phase of hot big bang, during which the expansion is decelerating. However, $\frac{1}{H^2a^2}$ has been reduced to a sufficiently small level at the onset of the hot big-bang phase that the subsequent decelerated expansion is compatible with an essentially flat and isotropic universe at the present time.

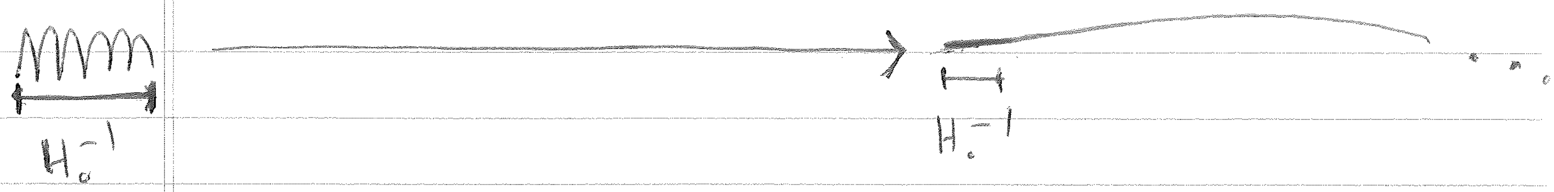
As a simple example of accelerated expansion, consider exponential expansion $a(t) \propto \exp(H_0 t)$ driven by a constant energy density $H_0^2 = \frac{8\pi G}{3} \rho_0$. During an exponential expansion $Ha \propto \exp(H_0 t)$. In order to increase Ha by a factor of $O(10^{30})$, required to solve the flatness problem, this phase needs to last for a period Δt such that $H_0 \Delta t \gtrsim 60$. This rapid expansion can take a very small patch, which could be flat or curved, and blow it up by such a huge factor that it will look essentially flat afterwards:



Moreover, this rapid expansion also implies that points that were apparently out of causal contact at the onset of the hot big bang phase actually emanated from a very tiny patch (blown up by the exponential expansion), and hence in causal contact:



This brief phase of an early accelerated expansion (prior to the hot big-bang phase) is called "cosmic inflation". It can provide a dynamical explanation to both of the flatness and isotropy problems. By the virtue of a very rapid expansion, it also erases any pre-existing inhomogeneity



Stretching the inhomogeneities by a factor $\gtrsim O(10^{30})$ will make regions of the horizon size H_0^{-1} look completely homogeneous after inflation.

This complete erasure of inhomogeneities poses a problem too. The level of inhomogeneity in the CMB is $O(10^{-5})$. On the other hand, even large initial inhomogeneities of $O(1)$ are reduced to ridiculously small values by inflation. Then the question is how to reconcile inflation (as a solution to the flatness and isotropy problems) with observation.

The answer is that inflation converts quantum fluctations of the vacuum into physical excitations, which will be the seed inhomogeneities observed in the CMB and later resulting in structure formation. This remarkable feature of inflation is what connects it to observation. Obtaining the correct value of fluctuations $O(10^{-5})$ is translated into a bound on a model of inflation that help determine its parameters. We will discuss inflation in more detail later on.

As a brief comment, inflation must end at some point, and the energy density that drove it must be converted into a thermal bath of elementary particles. This stage where transition from inflation to the hot big-bang universe occurs is called "reheating". At the end of reheating a radiation-dominated universe is formed and its subsequent evolution will follow that in the big-bang model.